

Math 6710 Applied Linear Operators and Spectral Methods Fall 2021 Syllabus

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Accessibility and Support: The preferred methods for reaching me are (in order of preference/reactivity): a message on Canvas or via email. I answer email daily, usually in the mornings.

Office hours: MW 11am-12pm, either in person (JWB 330) or using the Zoom handle <https://utah.zoom.us/j/93334775866> (passcode available in Canvas). I am available at other days/times upon request. Please check Canvas for any changes or cancellations.

Course Description:

This is an introduction to functional analysis with an eye on applications and is intended for graduate students that have had undergraduate real analysis (e.g. the Math 3210-3220 series or equivalent OR preferably Math 5210 or equivalent).

For mathematics graduate students:

To earn a “high-pass” in this class you need a **A**. To earn a “pass” you need a **A–** or **B+**. Earning a “pass” or “high-pass” in this class can dispense you from taking the preliminary / written qualifying exam associated with this class. For more details, please see the graduate bulletin, which is the authoritative document on this matter. Also do not hesitate to ask the instructor about this.

Course Details:

Course Type: In Person.

Location & Meeting Times: LCB 121. TuTh / 2pm-3:20pm.

Attendance & Punctuality: Attendance is expected. For those that cannot make it class notes will be regularly posted on Canvas. To get the best learning experience, you are encouraged to attend and actively participate in the lectures. Examples of participation are to ask questions, ask to re-explain or review some concept etc. . . Lectures should not be a monologue, participating is encouraged and makes the lecture more valuable for everyone involved.

Course Materials: All materials for this course (including class notes and problem assignments) are copyrighted. Do not distribute or share course resources without instructor permission.

Textbook: “Introductory Functional Analysis with Applications” by Kreyszig (Wiley Classics Library). ISBN-13: 978-0471504597. Price: about \$140 (as of Aug 2021).

Optional supplement: “Introduction to the Theory of Distributions” by Friedlander and Joshi (2nd edition, Cambridge University Press). ISBN-13: 978-0521649711. Price: about \$60 (as of Aug 2021)

Other references:

- “Functional Analysis, Sobolev Spaces and Partial Differential Equations” by Haim Brezis. ISBN-13: 978-0387709147.
- “Lecture Notes on Functional Analysis: With Applications to Linear Partial Differential Equations” by Alberto Bressan. ISBN-13: 978-0821887714.

Syllabus subject to change: This syllabus is meant to serve as an outline and guide for our course. Please note that I may modify it with reasonable notice to you. I may also modify the Course Schedule to accommodate the needs of our class. Any changes will be announced in class and posted on Canvas.

Content Overview: This class is an introduction to functional analysis. Most of the class focusses on how do results of linear algebra change when we are in infinite dimensions. To give you a taste of how interesting things get: in normed finite dimensional vector spaces we know that all closed and bounded sets are compact, however this no longer holds in infinite dimensions! We will work our way through, culminating with spectral theory, i.e. studying particular linear operators (matrices in finite dimensions) that are diagonalizable. We then use the tools of functional analysis to understand distributions or “generalized functions”.

Course expected learning outcomes: After this class, students should be familiar with the concepts outlined in the detailed syllabus. Students are expected to be able to apply these concepts to different situations, understand the proofs of such concepts and also be able to reproduce their derivation, with possibly some guidance. This is reinforced by proving results during regular homework problems and by applying the results to a variety of examples.

Course Design: Homeworks are assigned weekly. Although it may be easy to find solutions online, you will not learn anything from it. It is better figure the problems out on your own or with a group of other students. It may be unpleasant and frustrating to hit dead-ends, make mistakes and spend a lot of time on this, but it is the only effective way I know to learn mathematics and that is exactly how research feels like (most of the time). To help you, there will be two regularly scheduled office hours per week. The instructor is also available for meetings (in person or virtual) with prior notice. There will be two exams: a midterm and a final, and both will evaluate how well you know the concepts learned in class, how well you can apply them and whether you can prove certain results (perhaps with some guidance).

Evaluation: All homework assignments are to be submitted electronically using Gradescope. Please check the Canvas webpage for more info about this. In particular, this means you need a means of scanning your assignment (such as a scanner or smartphone) OR you may also handwrite your assignments on a tablet. Typing up your assignments is also acceptable but NOT necessary. The midterm and final are in person and on paper.

- **Homeworks – 50%:** Problems are assigned weekly and a (clearly indicated) subset of them will be graded. The two lowest homework grades will be dropped to compute the homework average.
- **Midterm – 20%:** 80min, in-person in our regularly scheduled classroom, closed book/notes, Thu Oct 7 2pm-3:20pm (tentative).
- **Final – 30%:** 120min, in-person in our regularly scheduled classroom, closed book/notes, Thu December 16 1-3pm (per university schedule).

Academic Code of Conduct: Students are encouraged to review the Student Code for the University of Utah: <https://regulations.utah.edu/academics/6-400.php>. In order to ensure that the highest standards of academic conduct are promoted and supported at the University, students must adhere to generally accepted standards of academic honesty, including but not limited to refraining from cheating, plagiarizing, research misconduct, misrepresenting one's work, and/or inappropriately collaborating. A student who engages in academic misconduct as defined in Part I.B. may be subject to academic sanctions including but not limited to a grade reduction, failing grade, probation, suspension or dismissal from the program or the University, or revocation of the student's degree or certificate. Sanctions may also include community service, a written reprimand, and/or a written statement of misconduct that can be put into an appropriate record maintained for purposes of the profession or discipline for which the student is preparing.

Grade scale: If X is your percentage grade, then $\{X \geq 93\% \Rightarrow A, X \geq 90\% \Rightarrow A-, X \geq 87\% \Rightarrow B+, X \geq 83\% \Rightarrow B, X \geq 80\% \Rightarrow B-, X \geq 77\% \Rightarrow C+, X \geq 73\% \Rightarrow C, X \geq 70\% \Rightarrow C-, X \geq 67\% \Rightarrow D+, X \geq 63\% \Rightarrow D, X \geq 60\% \Rightarrow D-, X < 60\% \Rightarrow E\}$. Letter grade assignments can be changed at the discretion of the instructor.

Communication

All course materials, such as lecture notes, assignments, solutions, grades, etc. will be posted on the Course Canvas site. Class announcements will be done via email through the Canvas server. You will be responsible for any information contained in them as well as the information announced in class.

It is your responsibility to also regularly check your Umail (make sure you set up forwarding if you do not check it regularly), your Umail is the only way for me to communicate privately with you, there will be occasions during the semester that we may need to reach out to you individually (e.g. regarding a grade or assignment) and it is in your best interest to respond promptly.

You may also consider installing the Canvas Student App on your smartphone to get notifications from this and other classes.

Feel free to contact me by email for questions at fguevara@math.utah.edu, I will do my best to answer emails promptly. I would like to encourage you to email me only if it is something personal that requires individual attention, if instead you have questions about logistics of the class, course material and assignments, and anything else your classmates may wonder as well, please post a question on the Discussions Board (in Canvas) instead. This way the information is shared quickly to the entire class, and each of you can benefit from seeing other classmates' questions.

Covid-19 considerations

University leadership urges all faculty, students, and staff to model the vaccination, testing, and masking behaviors we want to see in our campus community. These include:

Vaccination: Get a COVID-19 vaccination if you have not already done so. Vaccination is proving highly effective in preventing severe COVID-19 symptoms, hospitalization and death from coronavirus. Vaccination is the single best way to stop this COVID resurgence in its tracks. Visit mychart.med.utah.edu, alert.utah.edu/covid/vaccine, or vaccines.gov to schedule your vaccination.

Masking: While masks are no longer required outside of Health Sciences facilities, UTA buses and campus shuttles, CDC guidelines now call for everyone to wear face masks indoors.

Testing: If you are not yet vaccinated, get weekly asymptomatic coronavirus tests. This is a helpful way to protect yourself and those around you because asymptomatic individuals can unknowingly spread the coronavirus to others. Saliva based testing is available at alert.utah.edu/covid/testing

Self-Reporting: All of us, including faculty, students, and staff, must self-report if we test positive for COVID-19 via this website: coronavirus.utah.edu.

Additional Policies and Resources

The Americans with Disabilities Act: The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability & Access, 162 Olpin Union Building, 801-581-5020, disability.utah.edu. CDA will work with you and the instructor to make arrangements for accommodations. All written information in this course can be made available in alternative format with prior notification to the Center for Disability & Access.

Addressing Sexual Misconduct: Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a Civil Rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran's status or genetic information. If you or someone you know has been harassed or assaulted on the basis of your sex, office for equal opportunity and affirmative action including sexual orientation or gender identity/expression, you are encouraged to report it to the University's Title IX Coordinator; Director, Office of Equal Opportunity and Affirmative Action, 135 Park Building, 801-581-8365, oeo.utah.edu/contact-us or to the Office of the Dean of Students, 270 Union Building, 801-581-7066, deanofstudents.utah.edu. For support and confidential consultation, contact the Center for Student Wellness, 426 SSB, 801-581-7776. To report to police, contact the Department of Public Safety, 801-585-2677(COPS), police.utah.edu.

Campus Safety: The University of Utah values the safety of all campus community members. To report suspicious activity or to request a courtesy escort, call campus police at 801-585-COPS (801-585-2677). You will receive important emergency alerts and safety messages regarding campus safety via text message. For more information regarding safety and to view available training resources, including helpful videos, visit safeu.utah.edu

University Counseling Center: University Counseling Center: The UCC staff is committed to supporting the mental health needs of our campus community. Their phone number is 801-581-6826. Their hours are Monday-Friday, 8:00am-5:00pm. For after-hours emergencies, contact the 24/7 Crisis Line: 801-587-3000. More information is at counselingcenter.utah.edu.

Office of the Dean of Students: The Office of the Dean of Students is dedicated to being a resource to students through support, advocacy, involvement, and accountability. It serves as a support for students facing challenges to their success as students, and assists with the interpretation of University policy and regulations. To contact the Office of the Dean of Students, please email deanofstudents@utah.edu or call 801-581-7066. There is more information at deanofstudents.utah.edu.

Detailed syllabus:

1. Introduction to function spaces (Chapters 1 and 2)

Metric spaces: convergence, completeness
Continuity of functions
Separability
Contraction mapping principle
Vector spaces
Banach spaces
Compactness
 L^p spaces (without measure theory)

2. Linear operators (Chapters 2 and 4)

Linear operators
Bounded linear operators
Linear functionals
Dual spaces
Compact operators
Mention Hahn-Banach Theorem, Open Mapping theorem, Closed Graph Theorem and consequences
Weak and weak* convergence
Reflexive spaces
Fredholm alternative

3. Hilbert spaces (Chapter 3)

Inner product spaces
Orthogonal projections
Orthonormal sets
Linear functionals and bilinear forms
Riesz representation theorem
Lax-Milgram theorem (supplementary—not in text)
Adjoint operator
Fredholm alternative in Hilbert spaces

4. Spectral theory (parts of Chapters 7, 8, 9)

Resolvent and spectrum
Basic results for bounded linear operators
Spectral properties of compact operators
Bounded self-adjoint operators
Spectral theorem for compact self-adjoint operators
More general spectral representations

5. Distributions (supplementary text)

Spaces of test functions
Definition of distributions
Operations on distributions
Fourier transform and tempered distributions