

# Syllabus

## History and Philosophy of Mathematical Logic

### PHIL 3013: class # 18176

#### Designation: HF

#### Credit: 3 Units

**Instructor:** Dr. Chapman Waters

**Office:** CTIHB 247

**Office Hours:** Mon/Wed, 3-5pm

**Email:** chapman.waters@utah.edu

**Class Location:** GC 1780

**Class Time:** MWF 11:50am - 12:40pm

**Prerequisites:** none, but a previous course in logic would be a bonus.

### Course Description

This course will examine the history and philosophy out of which contemporary mathematical logic developed. Historical figures that will receive special emphasis include Cantor, Frege, Russell, Hilbert, and Gödel. Philosophical emphasis will be on issues pertaining to the metaphysical and epistemological foundations of mathematics and logic. For a more in depth characterization of topics covered, see “Learning Outcomes and Course Structure” below.

### Learning Outcomes and Course Structure

Students will gain an understanding of various issues that logicians are currently interested in by looking at the historical developments that gave rise to them. Equally important is that students will gain familiarity with the philosophical motivations and/or controversies that are intertwined with these developments.

Specifically, the course will be organized around the following figures and students will gain an understanding of their logical/mathematical/philosophical contributions:

#### *GEORG CANTOR, INFINITY, AND SET THEORY*

Set theory would have been a footnote in the history of mathematical logic, if not for Cantor’s amazing discovery of the hierarchies of larger infinities. We will examine this discovery, its history, and surrounding philosophical issues.

#### *FREGE, RUSSELL, AND LOGICISM*

This portion of the course will start with Gottlob Frege’s “Begriffsschrift” (1879), that is to say, the invention of modern quantificational logic. Particular points of focus will be on Frege’s understanding of the purpose of his notation (ie, as a scientific tool), Frege’s conception of the discipline of logic (ie, as the science that studies “the laws of truth”), and Frege’s attempt to establish Logicism (ie, the thesis that mathematics is a branch of logic). We will trace these ideas onto Bertrand Russell – who discovered the paradox that brought down Frege’s project – by considering Russell’s subsequent vindication of Logicism and the Ramified Theory of Types.

### *HILBERT'S PROGRAM*

With the paradoxes of set theory, and the subsequent troubles for Logicism, another important development came with David Hilbert. “Hilbert’s Program” provided the “formal” approach to axiomatic systems needed to overcome the paradoxes, account for the true nature of the infinite, and it provides the historical antecedent of the now widely accepted “model-theoretic” conception of logic.

### *GÖDEL'S INCOMPLETENESS THEOREMS*

We finish the course by considering Gödel’s Incompleteness Theorems. Gödel’s theorems establish that no final systematization of many important areas of mathematics is attainable. This groundbreaking achievement introduced new methods and problems for logico-mathematical investigation — and it set the agenda for projects which are still underway today. After gaining a rudimentary understanding of how Gödel proved his theorems, we will take up some of the issues that it raises: Does Gödel’s result have devastating effects on Formalism, Logicism, and other foundational programs in mathematics? Does it establish the truth of mathematical Platonism, the impossibility of genuinely intelligent machines (strong AI), or even the existence of God?!

### **Required Texts**

All readings will be available on the course Canvas page, in Perusall, or through the Marriot Library’s webpage.

### **Grading**

Grades will be given on a 0-100 point grading scale:

100-93:	A
92-90:	A-
89-87:	B+
86-83:	B
82-80:	B-
79-77:	C+
76-73:	C
72-70:	C-
69-60:	D
59-0:	E/F

Final grades will be determined as follows:

25%	Accountability Tasks
25%	Reading Discussions in Perusall
30%	Term Paper
20%	Comprehensive Final Exam

### *ACCOUNTABILITY TASKS*

There will be between 5 and 20 accountability tasks assigned over the course of the semester. These will take various forms, including (but not limited to) taking attendance, short pop-quizzes, group-work, in-class debates, short-reflection pieces on that day’s reading, among other

things. What accountability tasks have in common is that they will be given during class-meetings and students will not know on which days they will be administered. Moreover, it's possible that more than one accountability task will be assigned during a single class period. These assignments will not be difficult. I anticipate that most students who come to class prepared will be able to earn most or all of the points on any given task. The goal of these assignments is to motivate students to attend class and to come to class prepared.

#### *READING DISCUSSIONS IN PERUSALL*

Perusall is a “social reading platform” available to students via Canvas. It allows students to digitally annotate readings (with text, links, and embedded content). The goal is to make the process of figuring out what’s happening in class readings one that is collaborative, engaging, and (hopefully) fun. There will likely be between 7 and 9 of these assignments offered this semester — depending upon how fast we move. *Only those six with the highest scores will count toward your final grade. It is your responsibility to ensure that you’ve engaged with at least six different discussions by the end of the semester.* For more information about Perusall and these assignments, see the appropriate document on the Canvas course homepage posted under “IMPORTANT CLASS LINKS.”

#### *TERM PAPER*

There are three components to your term paper, together worth 25% of your final grade. More detailed instructions will be provided in class and on Canvas (including a grading rubric), but here is the basic format:

**Abstract (5%):** The abstract will include a thesis statement and 150 word summary of the paper you plan to write.

**Prospectus (10%):** Students will provide an outline of the content of their term paper, sketching the arguments, objections, etc., that they intend to provide. This should be no more than 1.5 or 2 pages in length (typed, single spaced). I will provide students with written feedback on these.

**Final Draft (15%):** All previous steps (1 - 2) are designed to help you to write the final version of your term paper successfully. Your main objective is to provide a reasoned defense of a claim that is philosophically relevant to course topics. The final version of the paper is to no longer than 8 or 9 pages, double spaced in MLA format, but may be shorter than this if it satisfies all relevant objectives.

#### *COMPREHENSIVE FINAL EXAM*

The final exam will consist entirely of multiple choice questions and will be comprehensive. Collectively, the questions will test your “bigger picture” understanding of course topics. There will likely be around 30 or 40 questions in total. A study guide is posted on Canvas which will be routinely updated throughout the semester.

#### **Course Policies**

**Class Meetings** — According to the University’s stipulations, this qualifies as an “in-person” course. In the absence of further developments relating to the evolving COVID-19 crisis, then, we will plan on meeting in-person at our regularly scheduled times. In-person classes will not be recorded nor will students have the option to attend class via online modalities. However, if

you need to seek an ADA accommodation to request an exception to this policy due to a disability, please contact the Center for Disability and Access (CDA). CDA will work with us to determine what, if any, ADA accommodations are reasonable and appropriate. Students who become quarantined due to a COVID-19 infection will receive accommodations on a case by case basis, provided that they are in contact with me about the situation in a timely manner.

Covid-19 Vaccinations – I \*strongly\* encourage any students who are currently unvaccinated to get vaccinated as soon as possible. While Utah law prohibits state universities from requiring COVID-19 vaccinations, it is highly effective in preventing severe COVID-19 symptoms, hospitalization, and death from coronavirus. Vaccinations are available to everyone 12 years and older. Appointments are open in the U of U Health system for patients as well as additional vaccine providers throughout Utah. For up-to-date campus vaccination information go to: <https://alert.utah.edu/covid/vaccine/>

Self-Reporting – Students *must* self-report if they test positive for COVID-19 via <https://coronavirus.utah.edu/> Students are encouraged to take advantage of university COVID-19 testing services: <https://alert.utah.edu/covid-19-testing/>. Voluntary asymptomatic testing will continue to be available weekly for all members of the campus community.

Canvas – The Canvas course webpage is your contact point for this course, and so, it's important that you familiarize yourself with it (for a tutorial page, visit the link pasted below). The “homepage” will provide a detailed course schedule that will be updated as we go. I will also use the Canvas homepage to distribute all course materials (course schedule, readings, handouts, study guides, etc.). Moreover, all coursework will be turned in through Canvas.

Canvas Tutorial: <https://community.canvaslms.com/docs/DOC-10701>

Attendance – Officially, I don't take it. But trust me when I say it is in your best interest to attend class regularly. A lack of attendance *will* be reflected in your final grade. If you have to show up late to class, that's fine. Come in when you can, but please do so quietly. It's better you show up late than miss an entire class.

Coursework Deadlines: In the absence of justifiable prior arrangements or cases of serious, unexpected and documented medical emergencies or bereavement, coursework must be turned in as scheduled. For circumstances that are not emergencies, students must send the instructor a request for an accommodation; this request should be sent a sufficient amount of time *before* the officially scheduled due date for the student to learn whether the excuse is acceptable. Requests sent after an assignment's deadline has passed will not be considered.

Academic Integrity: You are expected to do your own work on all assignments. The *minimum* penalty for cheating or plagiarism is a failing grade in the course. You will also be reported to the University's Academic Misconduct Committee. Students are responsible for knowing and understanding the University's policy on academic misconduct. For further information about this, consult the following website:

<https://regulations.utah.edu/academics/6-400.php>. Course Policies

### Course Schedule

Below, I've provided a general outline of the course schedule. However: *this syllabus is NOT a binding legal contract*. I reserve the right to revise this schedule (including due dates for assignments) as I see fit. If there are any changes, they will be reflected in the official "Course Schedule" on the CANVAS course page.

#### Weeks 1 - 3: CANTOR, INFINITY, AND SET THEORY

Historical introduction to Cantor's life and works  
 Introduction to elementary set theory  
 Infinity and Cantor's Transfinite Paradise  
 Axiomatic set theory in the twentieth century

#### Weeks 4 - 10: FREGE, RUSSELL, AND LOGICISM

Historical introduction to Frege's life and works: The Logician Project  
 Frege's *Begriffsschrift*: a revolution in logic  
 Frege's *Grundgesetze*: the definition of Number in purely logical terms  
 Function and Concept/Logic: The subject matter of logic  
 The *Grundgesetze* and Russell's Paradox  
 Russell's Resolution of the Paradox

#### Weeks 11 - 12: HILBERT'S PROGRAM

Historical introduction to Hilbert's Program: axiomatic systems and the demand for consistency  
 The Frege-Hilbert Controversy  
 Hilbert's Formalism, Model Theory, and Metalogic

#### Weeks 13 - 16: GÖDEL'S INCOMPLETENESS THEOREMS

Proofs of consistency for *Principia Mathematica* and  $L_q$   
 Gödel's proofs and its consequences

**Finals Week:** Monday 12/11 - Friday 12/15

**Final Exam Time:** Thursday December 14, 10:30am - 12:30pm

### Other Policies and Resources

#### **Problematic Historical Figure Statement**

Historical figures in philosophy, as well as many other intellectual domains, often held horrible views; views that, if held today, would be regarded as abhorrent. The fact is that many notable thinkers from history were racists, sexists, or any number of other terrible things. Some of the figures we will cover this semester are no exception. Fortunately, though, the abhorrent views of these particular figures are completely unrelated to their mathematical/logical/philosophical doctrines — and, in this class, we are exclusively interested in the latter. So, it'll be easy for us to consider only that which is valuable and leave the rest behind. Still,

it's important to acknowledge both the good and the bad in these figures — and that one's being an intellectual giant in one domain doesn't mean that one isn't a despicable, block-headed fool in another.

### **Veterans Center**

If you are a **student veteran**, I want you to know that the U of Utah has a Veterans Support Center on campus. They are located in Room 161 in the Oplin Union Building. Hours: M-F 8-5pm. Please visit their website for more information about what support they offer, a list of ongoing events and links to outside resources: <http://veteranscenter.utah.edu/>. Please also let me know if you need any additional support in this class for any reason.

### **Lauren's Promise: I will listen and believe you if someone is threatening you**

Lauren McCluskey, a 21-year old honor student and athlete, was murdered in October of 2018 by a man she briefly dated on the University of Utah campus. *We must all take measures to ensure that this never happens again.* Any form of sexual harassment or violence will not be excused or tolerated at the University of Utah. If you are experiencing sexual assault, relationship violence, or stalking, you may report it to me and I will connect you with the appropriate resources. If you are in immediate danger, call *911*.

### **LGBT Resource Center**

If you are a member of the **LGBTQ community**, I want you to know that my classroom is a safe zone\*. Additionally, please know that the U of Utah has an LGBT Resource Center on campus. They are located in Room 409 in the Oplin Union Building. Hours: M-F 8-5pm. You can visit their website to find more information about the support they can offer, a list of events through the center and links to additional resources: <http://lgbt.utah.edu/>. Please also let me know if there is any additional support you need in this class.

### **Learners of English as an Additional/Second Language**

If you are an English language learner, please be aware of several resources on campus that will support you with your language development and writing. These resources include: the Department of Linguistics ESL Program (<http://linguistics.utah.edu/esl-program/>); the Writing Center (<http://writingcenter.utah.edu/>); the Writing Program (<http://writing-program.utah.edu/>); the English Language Institute (<http://continue.utah.edu/eli/>). Please let me know if there is any additional support you would like to discuss for this class.

### **ADA**

The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability Services, 162 Union Building, 581-5020 (V/TDD). CDS will work with you and the instructor to make arrangements for accommodations. All written information in this course can be made available in an alternative format with prior notification to the Center for Disability Services.

### **Nondiscrimination**

The University of Utah does not discriminate on the basis of race, ethnicity, color, religion, national origin, sex, age, disability, sexual orientation, gender identity, gender expression, genetic information or protected veteran's status, in employment, treatment, admission, access to educational programs and activities, or other University benefits or services. Inquiries concerning perceived discrimination or requests for disability accommodations may be referred to the University's Title IX/ADA/Section 504 Coordinator in the Office of Equal Opportunity

and Affirmative Action, 135 Park Building, 801- 581-8365, or the Office of the Dean of Students, 270 Union Building, 801-581-7066.

### **Sexual Misconduct/Title IX**

Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a civil rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran's status or genetic information. If you or someone you know has been harassed or assaulted, you are encouraged to report it to the Title IX Coordinator in the Office of Equal Opportunity and Affirmative Action, 135 Park Building, 801- 581-8365, or the Office of the Dean of Students, 270 Union Building, 801-581-7066. For support and confidential consultation, contact the Center for Student Wellness, 426 SSB, 801-581-7776. To report to the police, contact the Department of Public Safety, 801-585- 2677(COPS).

### **Wellness Statement**

Personal concerns such as stress, anxiety, relationship difficulties, depression, cross-cultural differences, etc., can interfere with a student's ability to succeed and thrive at the University of Utah. For helpful resources contact the Center for Student Wellness at [www.wellness.utah.edu](http://www.wellness.utah.edu) or 801-581-7776.

### **University Safety Statement.**

The University of Utah values the safety of all campus community members. To report suspicious activity or to request a courtesy escort, call campus police at 801-585-COPS (801-585-2677). You will receive important emergency alerts and safety messages regarding campus safety via text message. For more information regarding safety and to view available training resources, including helpful videos, visit [safeu.utah.edu](http://safeu.utah.edu).

### **Philosophy Resources: (helpful for research and getting a better handle on complex issues)**

Stanford Encyclopedia of Philosophy: <https://plato.stanford.edu/>

Internet Encyclopedia of Philosophy: <https://www.iep.utm.edu/>